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The Stress-Strain Properties of Nitro-Cellulose and the Law of Its Optical Behaviour

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V. *The Stress-strain Properties of Nitro-cellulose and the Law of its Optical Behaviour.*

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THE physical characteristics of transparent bodies capable of resisting stress have been the subject of much investigation, and in particular the properties of various glasses* have been studied with much thoroughness since these latter have an extensive use both for commercial and scientific purposes.

In recent years many new forms of optical materials have found an industrial use, and especially nitro-cellulose compounds, which are valuable in cases where glass is not suitable.

The mechanical and optical properties of such bodies have not, so far, been examined in very great detail, and the present paper describes some experimental evidence which has been obtained and which it is hoped to extend as opportunity occurs, since this has an important bearing on the study of stress problems arising in engineering practice.

The principal matters which are examined in the present communication are the mechanical properties of nitro-cellulose under pure tensile and bending stresses and the laws of its optical behaviour under these kinds of stress. In the course of the experimental study a considerable number of specimens have been examined, all of which are the manufacture of the British Xylonite Company.

The salient features of the material are its great flexibility and toughness, and the ease with which it can be drilled, turned or machined. By suitable adjustment of the condition of nitration of the body the hardness of the material can be varied through a considerable range, but owing to the difficulties created by the stress of war it has not been possible to make this investigation cover materials possessing a great range of hardness, and in fact all the specimens taken were originally selected

* L. N. G. FILON, "On the Variation with the Wave-length of the Double Refraction in Strained Glass," 'Camb. Phil. Soc. Proc.,' vol. XI., Part VI.; vol. XII., Part I.; and vol. XII., Part V.; see also 'Phil. Trans.,' A, vol. 207, and 'Roy. Soc. Proc.,' A, vols. 79 and 89. F. POCKELS, "Über die Aenderung des Optischen Verhaltens verschiedener Gläser durch elastische Deformation," 'Annal. d. Physik,' 1902, and F. D. ADAMS and E. G. COKER, "The Cubical Compressibility of Rocks," 'Trans. Carnegie Institute.'

 [Published October 18, 1920.

on account of their transparency and freedom from initial stress. The sheets from which the specimens are made differ greatly in age, one has been in stock for at least eight years, most of the others have been stored one or more years.

In order to examine the stress-strain properties of this material it is unnecessary to use a very delicate extensometer as the value of the modulus for direct stress is comparatively small, and for the purposes of these experiments a very simple form is employed consisting of a pair of clips attached respectively to a scale and a pointer, which latter slides over the scale and is kept in contact with it by suitable attachments. In order to examine the optical properties of the material while under stress, both scale and pointer are perforated to give a window opening, and thereby permit a beam of polarised light to be transmitted through the specimen under examination. With this instrument and special magnifying devices it is possible to estimate extensions of 0·0002 inch.

A preliminary examination of the problem set out above may be described with reference to some experiments on a specimen which was originally used in 1911 for determining the stresses in a notched tension member.*

Two bars, each 1 inch wide, were cut, at that time, from a clear plate of xylonite $\frac{3}{16}$ inch thick, and each was fashioned with notches of different sizes along the edges. One of these specimens has been used for the present test. A length of 6 inches was used for observations of the longitudinal strains, while the lateral strains required for determining POISSON'S ratio $\frac{1}{m} = \sigma$ have been measured by aid of a strain-measuring apparatus having a unit reading of $\frac{1}{1,000,000}$ inches. Unless otherwise stated these latter measurements are in the direction of the thickness of the material. As the details of the measuring apparatus and the cylindrical recorder used with it have already been described† they are not referred to further here.

Longitudinal Extension.—The specimen was examined in the polariscope under a moderate load and it was found that the stress was very uniformly distributed over a length of $6\frac{1}{2}$ inches, but the remaining part of the parallel portion showed signs of unequal stress distribution owing to the enlarged ends. It was therefore marked off approximately into half-inch lengths over a total length of 6 inches, the exact distance being read to $\frac{1}{1000}$ inches.

YOUNG'S *Modulus.*—As it is convenient to start with a load of 20 lbs. on the specimen, a preliminary observation is made to determine the corresponding extension, and this value is allowed for in subsequent readings for convenience in plotting from a zero strain value.

* E. G. COKER, "The Effects of Holes and Semi-circular Notches on the Distribution of Stress in Tension Members," 'Proc. Phys. Soc.,' 1911.

† "Photo Elasticity for Engineers," by Prof. E. G. COKER, D.Sc., F.R.S., 'The Institute of Automobile Engineers,' November, 1917.

Six sets of observations are shown in fig. 1, the loads being applied in 20-lb. increments.

The maximum loads vary from 160 lbs. to 200 lbs., and after the strains reach their final value the load is again taken to its initial value and the scale read, to note the "semi-permanent set." The time interval between successive loads varies from one to

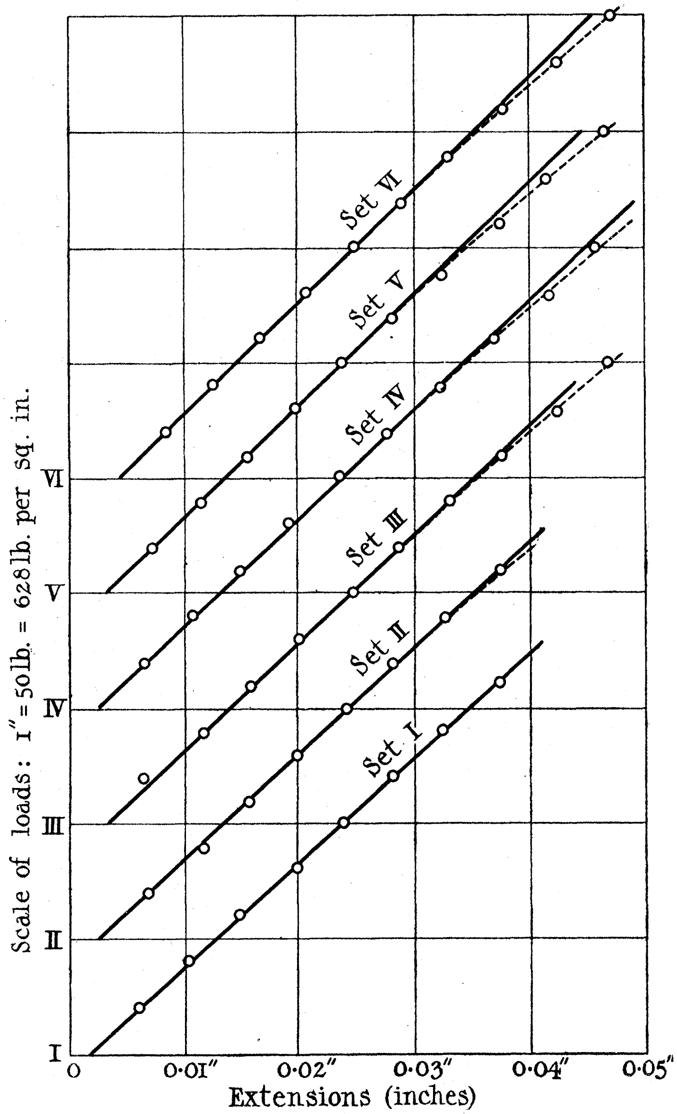

Fig. 1.

two minutes. It is found that the stress-strain readings so obtained are approximately linear, except at the highest values, but in order to obtain as correct a value of YOUNG's Modulus, E, as possible only measurements between 40 and 140 lbs. load are used, as the readings between these points are considered to be the most reliable. The strain corresponding to this difference of stress of 1256 lbs. per sq. inch is 0·00354 inch giving a value of E = 355,000.

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It will be observed that there is no very pronounced elastic limit, and that the curve is nearly straight up to 150 lbs. load (1900 lb./in.^{2}), which latter value may be taken as the elastic limit of the material. There is a "semi-permanent" set of 0·001 inches for each repetition of load, and a pronounced recovery between successive loadings especially with a short period of rest.

Measurements at Higher Stresses.—The spring balance used to measure the moderate loads in the above observations had a maximum capacity of 200 lbs., but for the higher stresses required, a balance recording up to 500 lbs. was necessary, the observations being made in a similar manner with readings on the magnified scale up to 400 lbs. load, and after this coarser readings were taken with the telescope. A maximum load of 476 lbs. (6000 lb./in.^{2}) was reached, but as the extension then increased very rapidly it was not possible to keep the load at this maximum value, moreover, as the stressing frame was of rather limited capacity for large strains, the test could not be carried to fracture, although a total extension of 1·211 inches was obtained. The observations also showed that the permanent extension was very uniform from section to section.

The condition of the material has in fact some resemblance to that of a mild steel which has been overstrained and allowed to rest. This is shown by a subsequent experiment in which the loading was repeated and the stress-strain properties examined anew. It was then found that the elastic limit of the material was still approximately at 150 lbs. load, corresponding to a stress of 1900 lbs. per sq. inch, but the modulus E had now risen to 502,000, measured in pound and inch units, as a result of the overstrain. The relations of load to extension for both conditions are shown in the accompanying fig. 2, but as the scales of load and extension are the same for both experiments the curve for overstrained material lies below that for unstrained material. It may be observed that the material possesses, in a marked degree, the property of contraction when the load is removed even when very much overstrained, and in this case when the full load of 300 lbs. was removed the semi-permanent extension was only 0·006 inch, and half of this disappeared with a few minutes rest.

Observations of lateral strain were also made with a suitable extensometer at several sections of the test bar, and their mean value for 100 lbs. load showed a strain of 0·00144, corresponding to a value of $m = \frac{1}{\sigma} = 2·45$ where σ is POISSON'S ratio.

The value of E is high as later experiments show, and this may possibly be due to an ageing effect, as in process of time the material appears to undergo some change, especially if the cut surfaces are not highly polished. This may probably be ascribed to the escape of a small portion of the volatile constituent of the material. It is also worthy of remark that the usual method of polishing appears to produce a thin outer layer which is harder than the interior, and this also has the effect of raising the value of E in thin specimens.

The effect of removing this thin layer of hard material has been under observation for some time, but in the experiments described here the flat sides are untouched, and the cut edges are unpolished although quite smooth.

Optical Properties.—There is considerable colour when an over-strained specimen is examined under no load in the polariscope. In the parallel part the colour is very uniform, and by comparing this with a previously unstrained specimen under load it appears that the permanent colour indicates a complex state of stress, since it could not be completely neutralised by a comparison tension piece, nor by bending the strained bar itself.

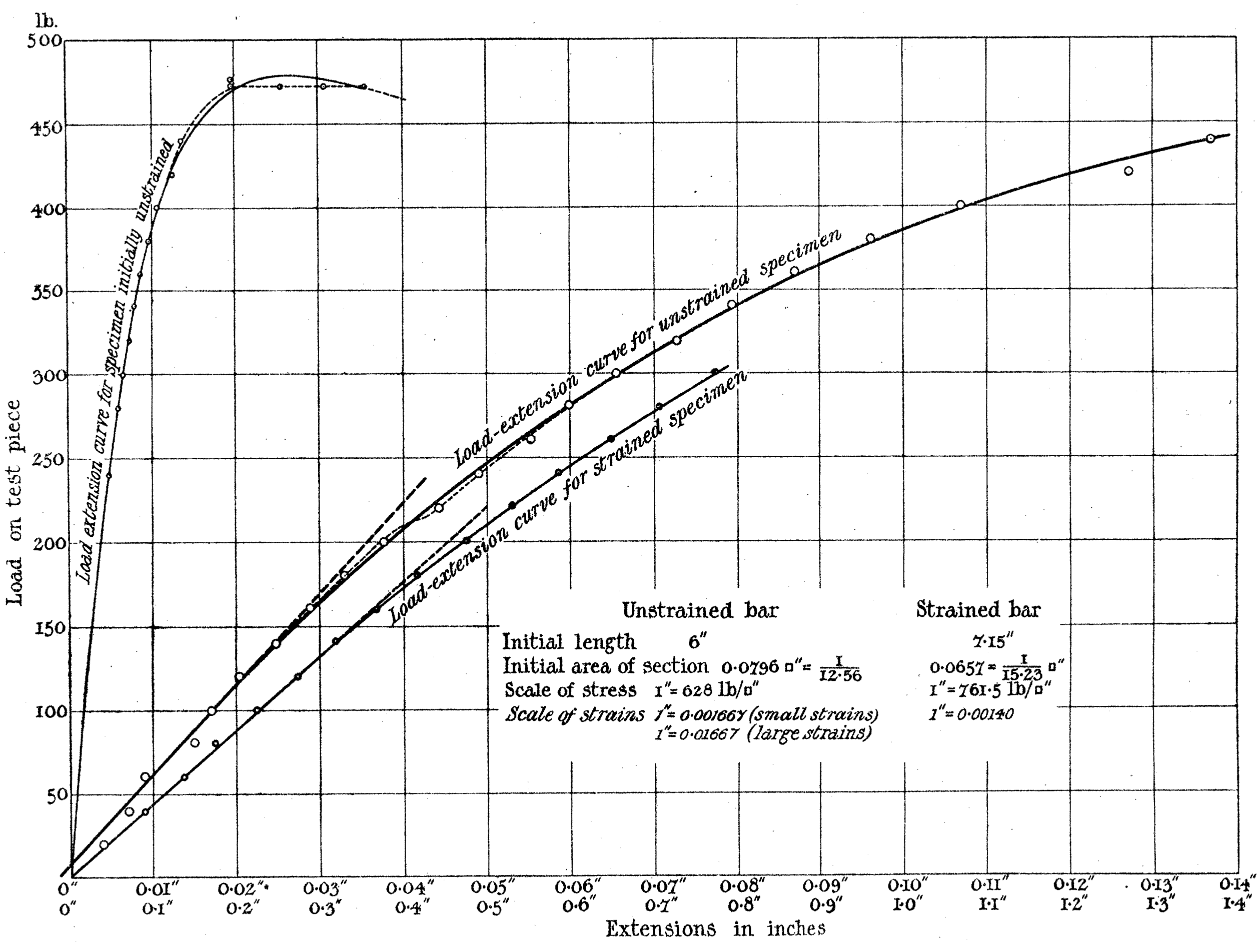


Fig. 2. Load-extension diagrams.

It may also be noted that nitro-cellulose with good optical properties is not apparently procurable, above $\frac{1}{4}$ inch thick, and it is difficult therefore to conform to the laws of similarity for the test specimens used in this investigation.

In later experiments the stress-strain properties of the material are examined both below and above the elastic limit, and the values of Young's modulus and Poisson's ratio are measured for a number of specimens of different thickness and varying age. Especial attention is also directed to test the validity of the stress-optical law of this material since this is a matter of fundamental importance and little attention has so far been given to it.

The result of breaking the first three specimens showed that the extensometer arrangement was defective. The tiny indentations at the sides, where the extensometer was attached, so weakened the specimens that they all broke at one or other of these sections. The thinnest one naturally was affected most, so much so that it fractured with very little extension. This defect was partly corrected by cementing small fillets on to the specimen from which the extensometer clips were supported, but in spite of this some of the specimens fractured outside the gauge limits, showing that even in a ductile material the effects of enlarged ends prevents equalisation of stress near the change of section under any condition of load.

Stress Optical Determination.—It was originally intended to study the stress-optical properties of nitro-cellulose by analysing the light which traversed the material by means of a spectroscope; but the necessary apparatus was rather difficult to procure, and it was convenient therefore to commence with a standard nitro-cellulose beam and use this for comparison with the optical phenomena observed in tension. The methods adopted here proved to be exceedingly well adapted for measurement of stress distribution beyond the elastic limit and are likely to be of great use hereafter. The comparison beam used is of rectangular section and is subjected to pure bending moment of known amount, and the stress at any point can therefore be calculated from the formula $f = \frac{My}{I}$ without appreciable error.

It is generally assumed that the relative retardation of the polarised rays in a piece of optical material under moderate stress is proportional to the difference of principal stresses at the point, but this may not be correct and cannot be assumed to hold without experimental proof. Hence the stresses in the comparison beam are restricted to small values, so that the limit of proportionality of stress to strain is not passed in order to give an opportunity of examining the possibility of the law following a linear strain function or possibly some more complex variable. In order to make the retardation in such a beam sufficiently great to balance the retardation in the highly stressed specimens, the thickness of the beam should be large. This is most conveniently obtained by placing the several beams side by side, with their ends clamped and pinned together, as shown in fig. 3 in which several beams are so fastened together by plates A, to which extension levers B are also attached for supporting loads C depending from hangers D. This compound beam is supported on knife edges two inches apart, and when loaded has its central section sufficiently removed from the supports to give pure bending moment at the central section. The material of the beams is almost perfectly elastic up to and probably beyond 1600 lb./in.^{2}, but they are actually not stressed to more than 1300 lb./in.^{2}. In some cases as many as eight beams $\frac{1}{8}$ inch thick are used in this way, and a strong beam of light is then necessary to enable a comparison to be made with the tension member under observation. A carbon arc is then used as the source of light, but when only two or three thicknesses are employed the light from a Nernst lamp is sufficient, but in all cases the images are observed directly by eye instead of projecting on to a screen. The general arrangement of the apparatus is shown in the accompanying fig. 4, in which a plane polarised beam of white light from a NICOL's prism A is transmitted through the tension specimen B, to which an extensometer C is secured, and is then focussed by a lens D on a horizontal slit in order that the light passing through the comparison beams

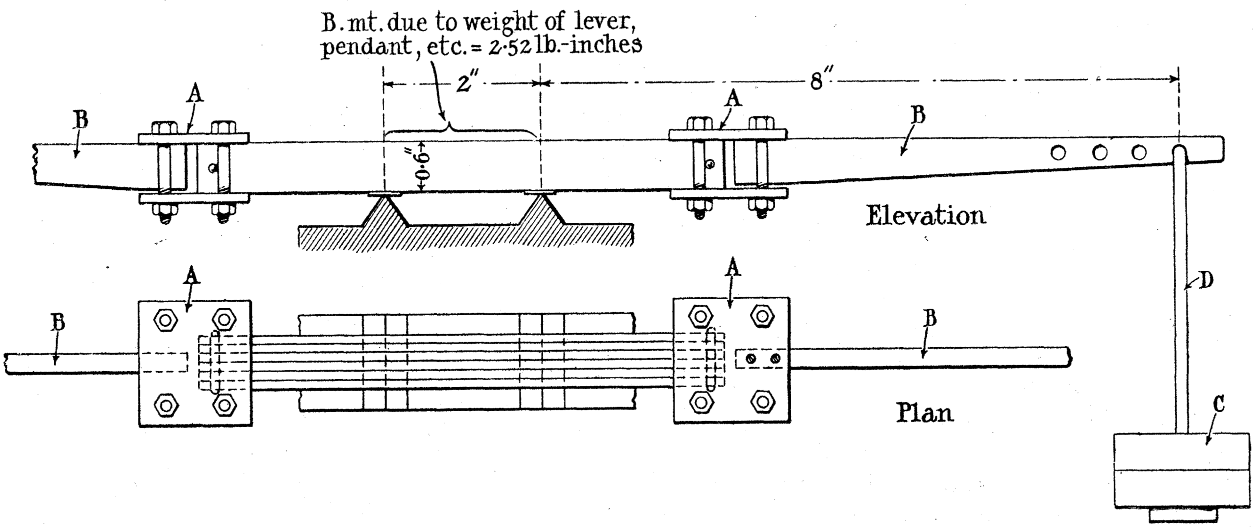


Fig. 3. Beam Comparator.

shall be at the same level throughout. This thin pencil of light is again brought to parallelism before passing through the compound beam F and analyser G, and is finally focussed on a ruled glass slide H provided with an eye-piece J. The weight of the extension beams and hangers causes a bending moment in the beams which has been allowed for in all calculations of stress. In order to compare the different

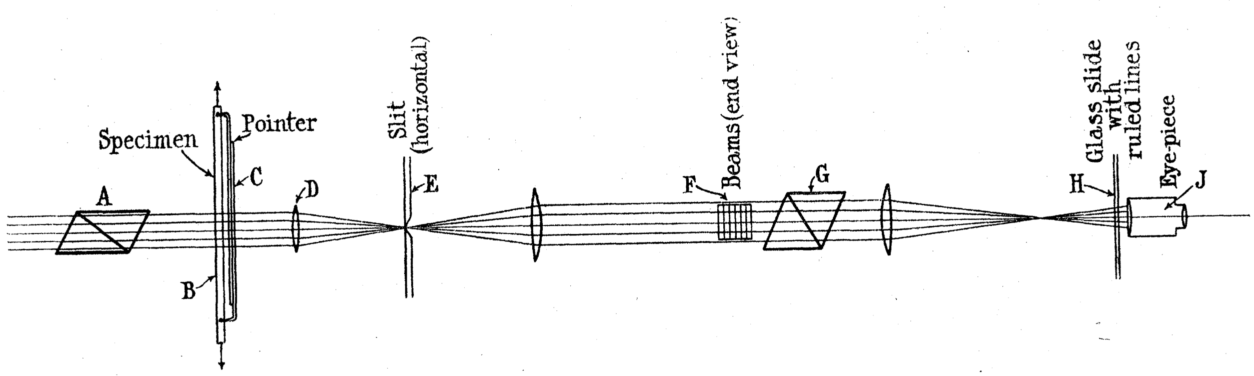


Fig. 4. Diagram of polariscope.

specimens one with another, an "equivalent stress" in each specimen is calculated, that is, such a stress as would produce the same relative retardation in a piece of nitro-cellulose of the same material as the standard beam, but of the thickness of the specimen under observation. Thus if the thickness of specimen is t and the stress in the beam at the points where the colour in the specimen is neutralised is f_0 and the corresponding thickness is t_0, we have the equivalent stress in the specimen $\frac{f_0 t_0}{t}$.

Now if M is the bending moment in the beam, d is its depth, and y is the distance from the neutral axis, then

$$f_0 = \frac{\mathrm{M}y}{\mathrm{I}} = \frac{\mathrm{M}y}{\frac{1}{12}t_0 d^3} = \frac{12\mathrm{M}y}{t_0 d^3},$$

so that the equivalent stress

$$f = \frac{f_0 t_0}{t} = \frac{12\mathrm{M}y}{td^3}.$$

Although as stated above the law of optical retardation is generally assumed to follow a linear law of stress difference, yet there is no apparent reason why it should not follow some other law, as for example a linear strain law or possibly contain terms involving squares of stress or strain. Some attempt has been made to find if the latter assumptions have any foundation, but if so the effects are within the limit of experimental error, and too small to be of any significance with the effect produced by a linear relation.

As regards the question whether this relation should be expressed in terms of stress or strain, it may be pointed out that an attempt is made here to test this with materials under direct stress, and that the validity of the law for combined stresses and strains still remains for consideration (apart from lateral strains, which are presumed to have no effect beyond altering the length of the path in which retardation takes place), but as in this case, if the standard, not stressed beyond the elastic limit is compared with another in which this condition is passed the experiments do in fact provide a means of discrimination, since in the standard, stress and strain are proportional, but are not so in general for the tension member. Hence if the form of the law of optical effect is assumed in terms of stress it does not exclude the possibility of finding from the experimental evidence whether it should not be expressed in terms of strain. We may, therefore, without loss of generality, take as an assumption the usual relation that relative retardation $\mathrm{R} = \mathrm{C}\,(\mathrm{P}-\mathrm{Q})\,t$ as a convenient expression where $(\mathrm{P}-\mathrm{Q})$ is the difference of principal stress $= f$, t is the thickness of the material and C is the stress-optical coefficient.

Let C_0 be the stress-optical coefficient of the standard beam. Then $\mathrm{R}_0 = \mathrm{C}_0 f_0 t_0$ for this beam.

When this latter is used to neutralise the retardation R in the specimen, since $\mathrm{R} = \mathrm{R}_0$, we have

$$\mathrm{C}ft = \mathrm{C}_0 f_0 t_0$$

But $t = t_0$, here and therefore

$$\mathrm{C}f = \mathrm{C}_0 f_0.$$

Now in general there is an initial retardation which is independent of any load. Let this correspond to stresses F, F_0. Then the condition $Cf = Cf_0$ becomes

$$C(f+F) = C_0(f_0+F_0)$$

or differentiating,

$$C \cdot df = C_0 \cdot df_0,$$

therefore

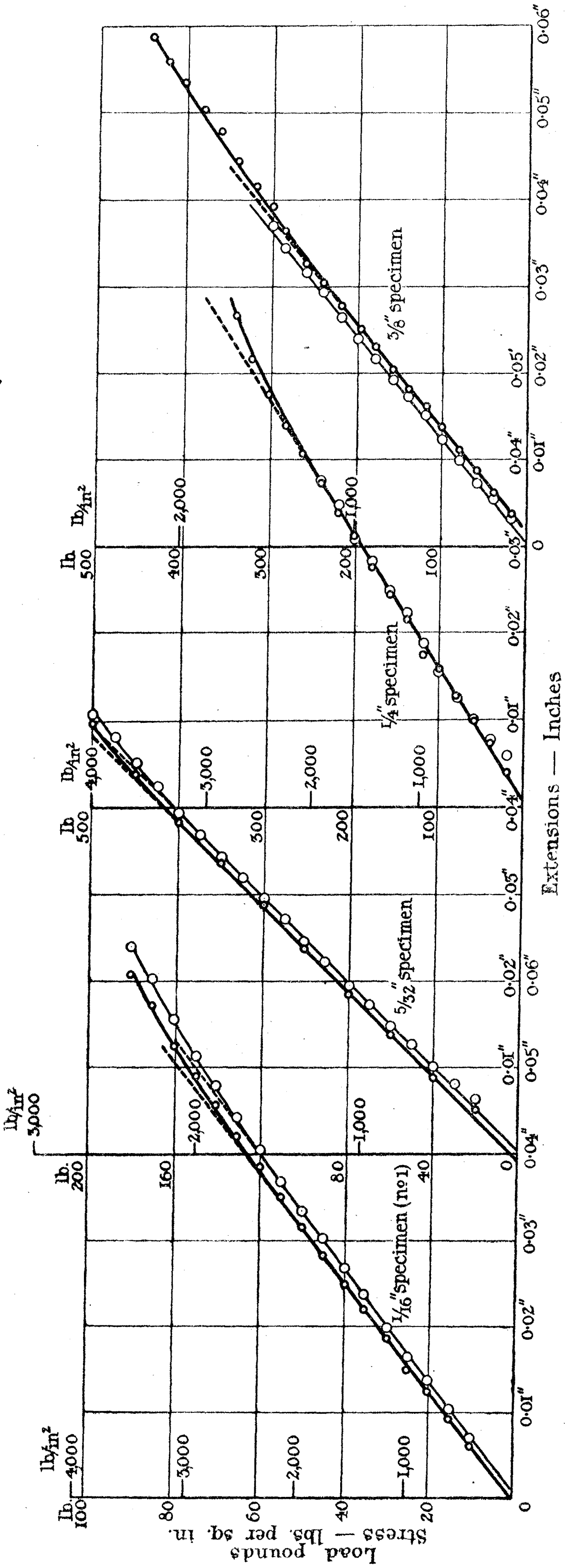
$$\frac{C}{C_0} = \frac{df_0}{df} = \frac{1}{\dfrac{df}{df_0}} = \text{reciprocal of slope of the stress/equivalent stress,}$$

which affords a convenient relation for examining the experimental data.

Turning now to the further experimental data upon the stress-strain properties of nitro-cellulose in tension a number of experiments have been made upon material of varying age and thickness, and these are plotted in the accompanying fig. 5, to show their characteristic properties under loads which sometimes exceed very considerably the elastic limits of the material.

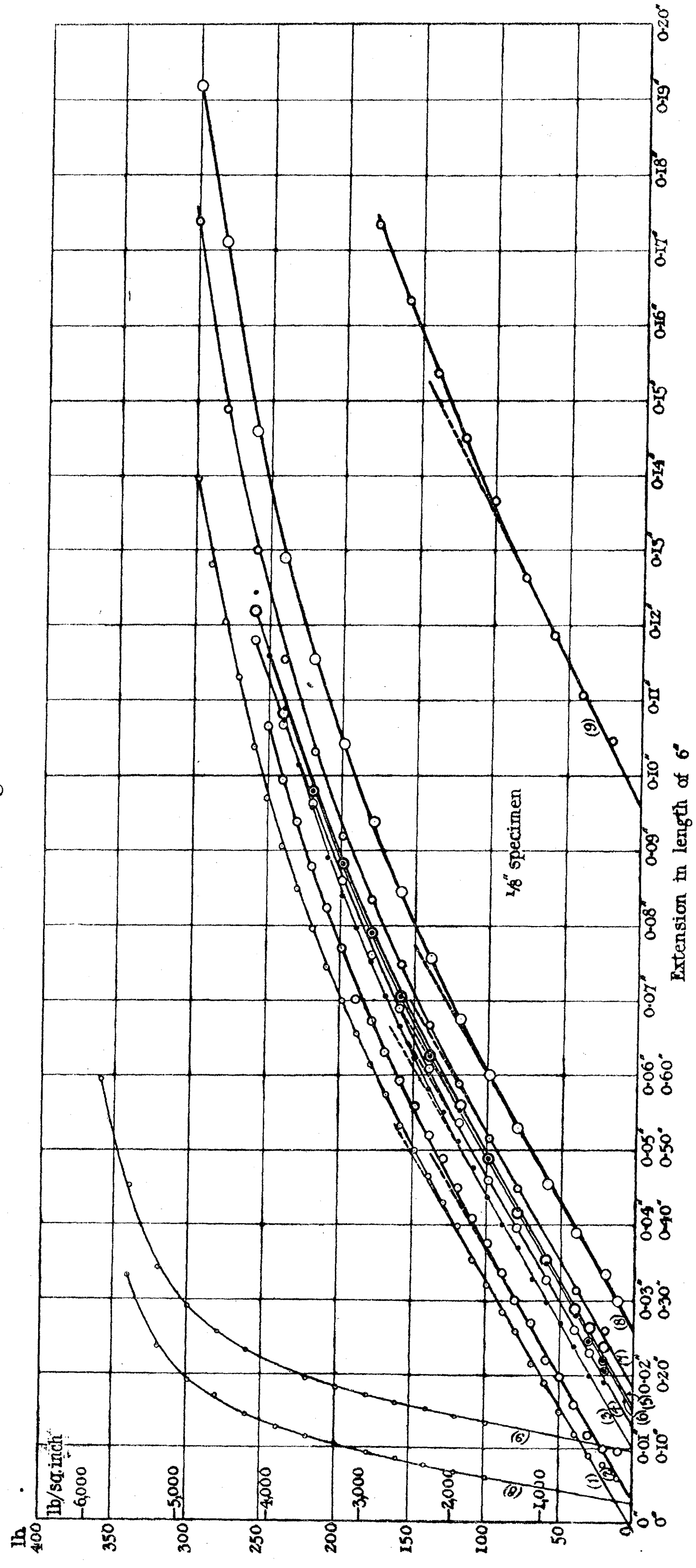
With the thinnest specimens $\frac{1}{16}$ inch thick it was not found possible to obtain a reliable value of POISSON'S ratio, but YOUNG'S modulus, E, has been found, and the measurements plotted in fig. 5 show a characteristic feature that, although the first test is carried well beyond the elastic range, as soon as the load is removed a total extension of 0·0608 inch is reduced to 0·0070 inch or only 0·001 inch more than obtained at the commencement of this test. Moreover the value of the modulus changes less than 2 per cent. under these circumstances due to the earlier loading. It is also large as the skin effect is pronounced. These general characteristics are also observable in the measurements recorded in this figure for much thicker material if due allowance is made for the diminished effect of the surface layers. The capacity of returning to its original shape after high loads is still more marked in the next series of experiments on material $\frac{1}{8}$ inch thick, fig. 6, in which a stress of nearly 5000 lbs. per sq. inch is reached in the first experiment (curve 1) with nearly complete recovery, and when further loads with maxima varying from 4000 to 5000 lbs. per sq. inch are applied (curves 2 to 8) these give almost identical values of YOUNG'S modulus on the straight part of the curve until the ninth loading, where there is a sudden fall to E = 261,000 with an extension of 0·1045 inch corresponding to the initial load of 20 lbs. After this, with the considerable initial extension of 0·4290 inch, there is a great rise in the modulus. The value of POISSON'S ratio is very constant and is here found to reach the highest value of $m = \frac{1}{\sigma} = 2{\cdot}7$.

Succeeding experiments on still thicker material confirm these results, and with the exception of the $\frac{3}{8}$-inch plate, the load extension curves agree in their linear character up to about 2000 lbs. per sq. inch, although the specimens differ in age and possibly also in composition. They have, however, the common feature of possessing



Extensions — Inches

Fig. 5.



Extension in length of 6"

Fig. 6. Load-extension diagram—low stresses

excellent optical properties and freedom from initial stress. The thickest plate, however, is exceptional, as its optical properties are poor.

Further experimental work on these materials is almost entirely devoted to the examination of the optical law of retardation under load, and for convenience all the data which follows is expressed as a stress or a strain, the units being pounds and inches, in which e is the strain under direct stress f, and the equivalent stress f_0 is obtained from the comparator beam. A typical example of these values is given in Table I., for material $\frac{1}{4}$ inch thick, as these measurements are referred to later for comparison with values of stress and strain obtained from spectrum observations.

TABLE I.

$\frac{1}{4}$-inch.			$\frac{1}{4}$-inch.		
e.	f.	f_0.	e.	f.	f_0.
0	0	73	0·0123	3345	3440
0·0003	160	220	0·0135	3505	3560
0·0007	319	322	0·0142	3665	3900
0·0013	478	440	0·0153	3825	4140
0·0020	637	660	0·0172	3980	4500
0·0027	797	880	0·0187	4140	4770
0·0033	956	1025	0·0208	4300	5100
0·0037	1115	1173	0·0212	4460	
0·0043	1274	1320	0·0237	4620	
0·0050	1433	1495	0·0287	4780	
0·0055	1594	1642	0·0320	4940	
0·0058	1752	1760	0·0370	5100	
0·0063	1912	1910	0·0553	5260	
0·0070	2070	2050	0·120	5420	
0·0077	2230	2200	0·157	5580	
0·0082	2390	2350	0·183	5740	
0·0087	2550	2540	0·247	5900	
0·0095	2710	2310	0·280	6055	
0·0102	2865	2860	0·340	6215	
0·0108	3025	3080	0·357	6375	
0·0117	3185	3230			

In the earlier experiment on the material $\frac{1}{16}$ inch thick, a fracture was obtained near the change of section and before the full extension developed, but still very nearly at the full load. It is included here (fig. 7) as, although the later parts of the stress-strain curves are not entirely satisfactory, this does not affect the problem in hand, since the stress-strain curve is not required very much beyond a pronounced yield in the material.

As a purely mechanical problem, however, there is a considerable amount of interest attaching to the accurate measurement of stress and strain over the whole of the plastic region, and it may be worth while at some future time to examine this with some care, especially if optical methods are applied to study the distribution of stress in purely plastic materials.

The stress-strain curves obtained for this thin material show a divergence from a linear law above 2000 lbs. per sq. inch whether plotted from the direct load or the optical stress measurements, but if the direct stress is plotted against its optical equivalent there is a definite linear law extending up to at least 4500 lbs. per sq.

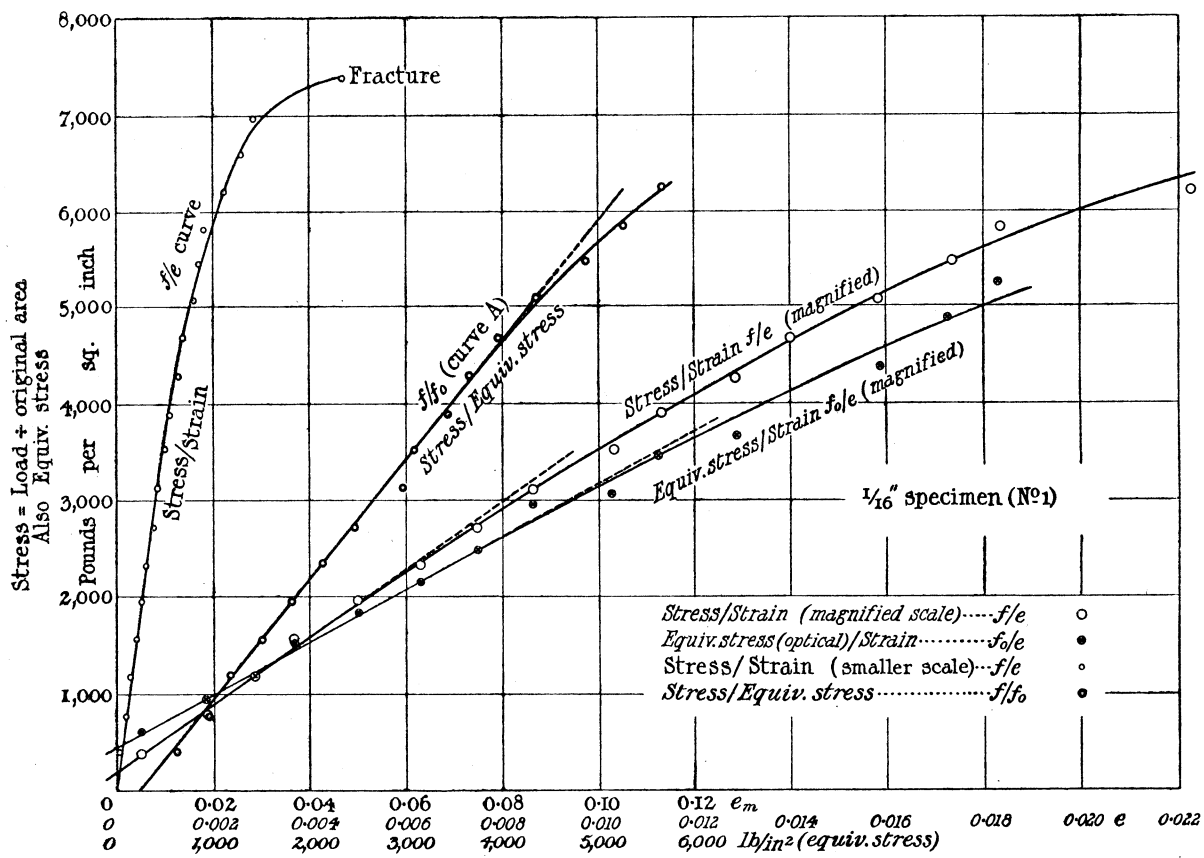
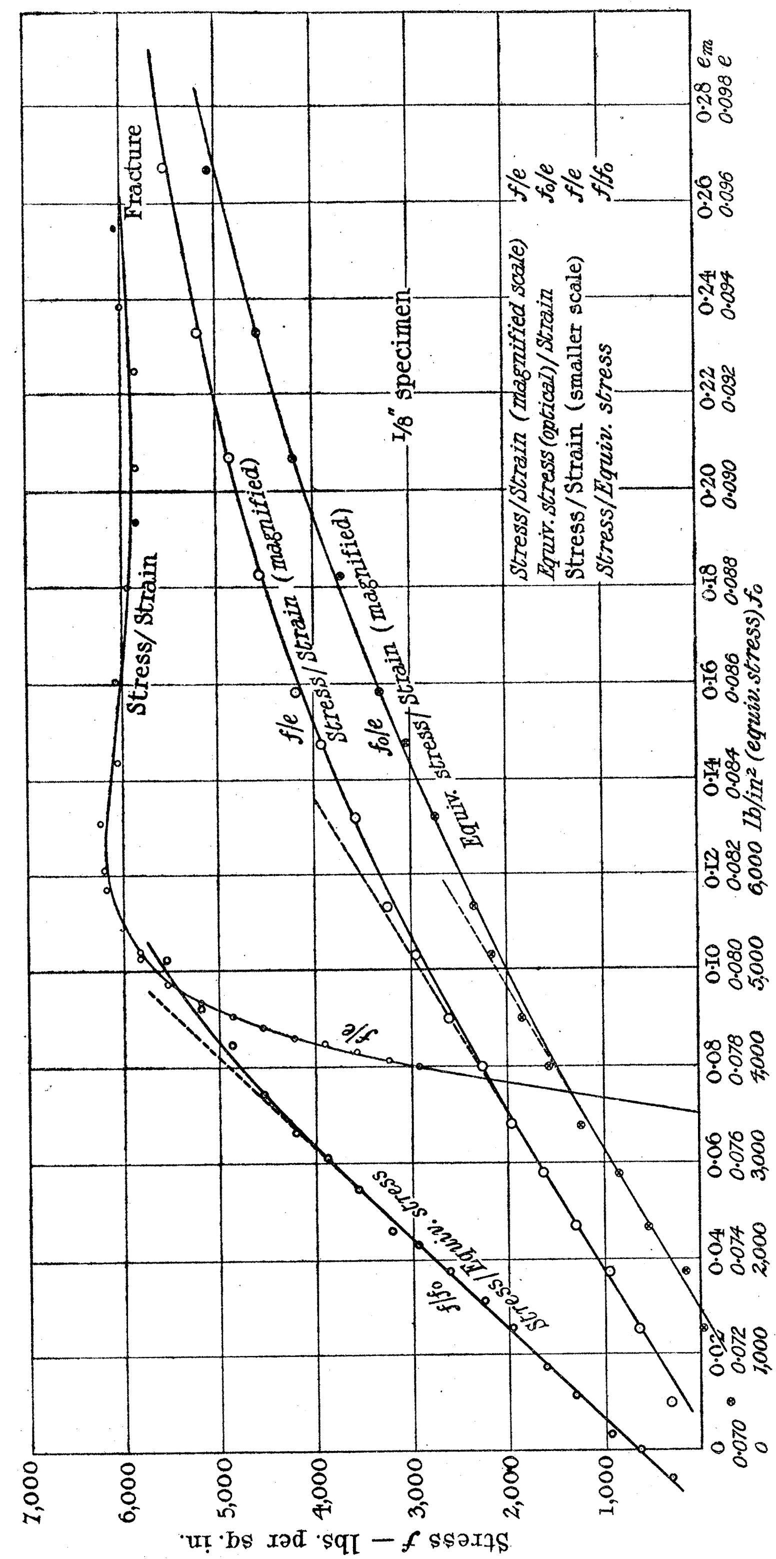


Fig. 7. Stress-strain curves of nitro-cellulose.

inch, and with only a small divergence at 5000 lbs. per sq. inch. The results in fact go to show that the law of retardation is linear as regards stress not only up to the elastic limit but actually to at least twice this range, where it is quite impossible for the strain to be linear. This result is shown in all the experiments on good optical material. Thus in plates $\frac{1}{8}$ inch thick where the elastic limit appears to be about 2250 lbs. per sq. inch, fig. 8, the corresponding value for f/f_0 shows no divergence from linearity until nearly double this amount, although the curve of f_0/e ceases to be linear at about 2500 pounds per sq. inch.



Stress f — lbs. per sq. in.
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0 0·02 0·04 0·06 0·08 0·10 0·12 0·14 0·16 0·18 0·20 0·22 0·24 0·26 0·28 e_m
0·070 0·072 0·074 0·076 0·078 0·080 0·082 0·084 0·086 0·088 0·090 0·092 0·094 0·096 0·098 e
0 1,000 2,000 3,000 4,000 5,000 6,000 lb/in² (equiv. stress) f₀
Fracture
Stress/Strain
f/e Stress/Strain (magnified)
f₀/e Stress/Strain (magnified)
Equiv. stress/
1/8" specimen
f/e
f/f₀ Stress/Equiv. stress
Stress/Strain (magnified scale) f/e
Equiv. stress (optical)/Strain f₀/e
Stress/Strain (smaller scale) f/e
Stress/Equiv. stress f/f₀

Fig. 8.

Somewhat similar results are obtained on plates $\frac{3}{16}$ inch thick, but in both experiments, fig. 9, the curves of f_0/e have a rather higher linear limit than the corresponding f/e curve, but here again the ratio f/f_0 is still linear to about the same range as in previous cases.

The case of plates $\frac{1}{4}$ inch thick, fig. 10, is more especially interesting from the fact that the stress-strain curve there shown is, at a later stage, obtained entirely from the optical effects observed from an analysis of the spectrum of a beam under uniform bending moment. It is sufficient to remark here that the f/f_0 curve shows a somewhat lower limit of linearity, although both the other curves have corresponding limits of 2000 lbs. per sq. inch.

When these curves are corrected for the change of cross-section which occurs as the test proceeds it is found, as fig. 10 shows, that the stress-strain curve f/e is perceptibly raised beyond the elastic limit and therefore tends more towards linearity, and the equivalent stress/strain curve is lowered and diverges still more from the linear relation. The stress/equivalent-stress curve has therefore a somewhat higher linear limit when this correction is made. Owing to the defective optical properties of still thicker material it was not found possible to examine these relations in a $\frac{3}{8}$-inch plate in a satisfactory manner.

Fracture.—The behaviour of nitro-cellulose at fracture is somewhat unusual for so ductile a material. As the load increases the section diminishes very uniformly at all parts removed from the enlarged ends, but there is little or no local contraction at any stage, and even at the fractured section, the cross-section differs but little from that at any other part of the bar, but after fracture there is a remarkable contraction in the total length accompanied by uniform expansion of the cross-section. This is shown in Table II., which gives a summary of the observations made and, except for one of the thin specimens and for the reasons given earlier, there is a recovery in length of from 6 to 9 per cent. after fracture. Various other measurements already described above are recorded here for convenient reference and also some ratios of the optical constants.

Spectrum Analysis of the Stress in a Beam.—The results of the optical examination appear to show the truth of the optical stress law for simple stress well beyond the elastic limit of the material, but the importance of this fundamental law makes it desirable to examine the matter in an independent way and possibly by a more rigid test than a comparison beam affords. An investigation of the optical phenomena presented by a beam under pure bending moment was made therefore on a rectangular strip $22\frac{1}{2}$ inches long, 1·005 inch deep and 0·2542 inch thick. Its specific gravity was approximately 1·361, this latter being determined at a temperature of 64° Fahr. by measurement of its volume and weighing in air.

The beam is supported as before on knife edges 2 inches apart, and the loading is applied at each end by dead weights having an overhang of $9\frac{3}{4}$ inches.

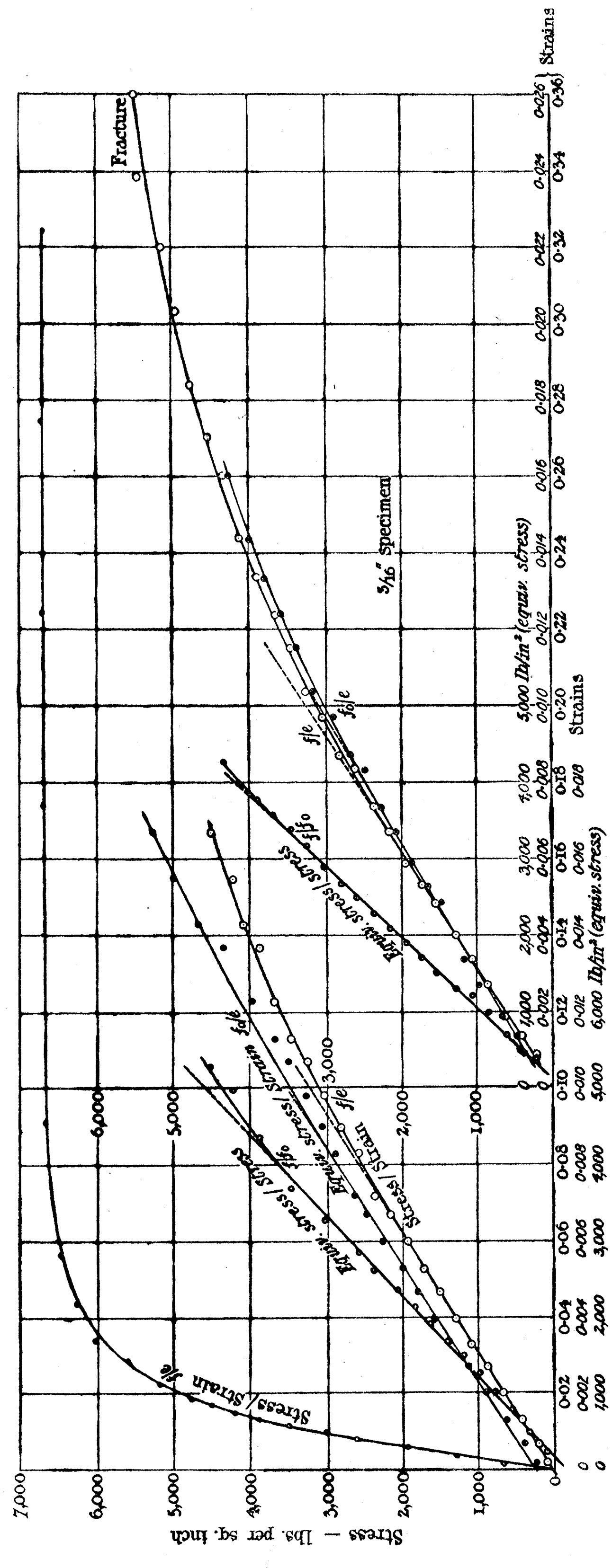

Stress — lbs. per sq. inch
Strains
Fracture
5/16" specimen
Stress/Strain f/e
f/e
f₀/e
f/f₀
Stress/Strain
Equiv. stress/Strain
Stress/Strain
Equiv. stress/Stress
Lb/in² (equiv. stress)

Fig. 9.

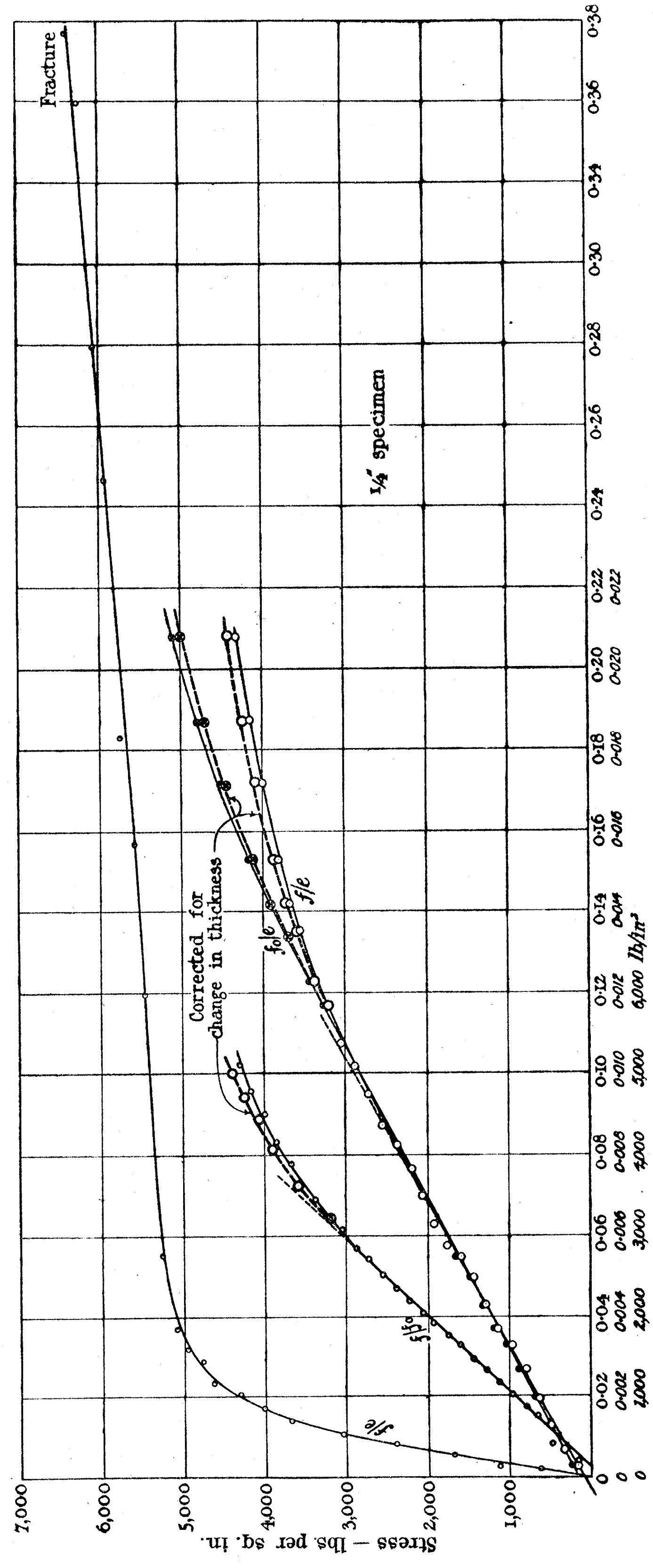

Stress — lbs. per sq. in.
Fracture
1/4" specimen
Corrected for change in thickness
f/e
f₀/e
f/f₀
Lb/in²

Fig. 10.

TABLE II.—Properties of the Specimens Tested.

Test length, 6-inch.

Specimen	$\frac{1}{16}''$ (No. 1)	$\frac{1}{16}'$ (No. 2)	$\frac{1}{8}''$	$\frac{5}{32}''$	$\frac{3}{16}''$	$\frac{1}{4}''$	$\frac{3}{8}''$
Width	0·5086″	0·5090″	0·5050″	0·4992″	0·4978″	0·4972″	0·4992″
Thickness	0·0505″	0·0506″	0·1213″	0·1517″	0·1863″	0·2533″	0·4045″
Area (in.^{2})	$0{\cdot}0257=\frac{1}{38{\cdot}85}\square''$	$0{\cdot}0258=\frac{1}{38{\cdot}75}\square''$	0·0613 □″	$0{\cdot}07575=\frac{1}{13{\cdot}2}\square''$	$0{\cdot}0928=\frac{1}{10{\cdot}76}\square''$	$0{\cdot}1258=\frac{1}{7{\cdot}97}\square''$	$0{\cdot}202=\frac{1}{4{\cdot}96}\square''$
Modulus $\frac{\text{lb.}}{\text{in.}^2}$	362,000	355,000	295,000	324,000	313,000	309,000	251,000
POISSON'S ratio	—	—	2·7	2·5	2·6	2·3	2·4
$\frac{C}{C_0}$ [optical]	0·82	0·84	0·97	1·00	1·00	0·95	1·20 (?)
Maximum extension before fracture	0·28″	1·66″	1·53″	2·14″	1·94″	2·14″	2·26″
Ultimate extension (after fracture)	0	1·23″	1·07″	1·67″	1·54″	1·72″	1·74″
Recovery in length	0·28″	0·43″	0·46″	0·47″	0·40″	0·42″	0·52″

The optical arrangements are modified for the new conditions as shown in the accompanying fig. 11. Light from the filament A of a Nernst lamp is focussed on to a vertical slit S by aid of a lens B, after passing through a NICOL'S prism M and this narrow band is in turn focussed on the central section of the beam at D, and analysed by a second NICOL'S prism N. A lens E placed at a convenient distance from the beam transmits this light as a parallel beam to a reflecting prism F, from which it passes through prisms G, H. The spectrum so obtained is focussed on a glass screen L ruled with lines $\frac{1}{100}$ inch apart, and provided with a micrometer eye-piece for measuring the ordinates of the bands observed.

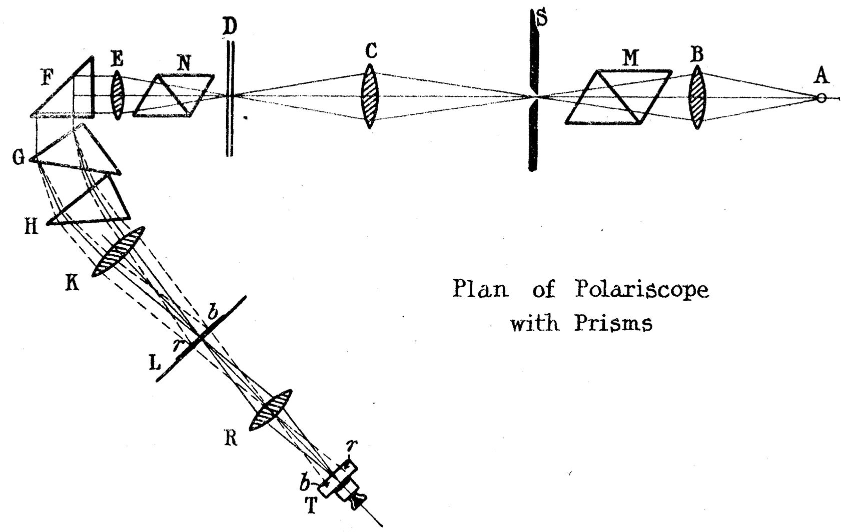


Fig. 11.

The field of view consists therefore of the spectrum of a Nernst lamp filament to which is added the effect produced by a narrow section of a beam of rectangular cross-section under pure bending moment. The relative retardation, owing to this latter stress effect, produces black bands in the field having a variable distance apart depending on the optical law of the retardation of the wave-length.

The general disposition of the field of view is shown in the accompanying fig. 12 in which bands of the first and second order appear on each side of the neutral axis C of the beam, and their co-ordinates are measured by reference to the graduations on the glass scale with the aid of a pair of parallel wires D, the positions of which can be adjusted vertically by a micrometer head E reading to $\frac{1}{6000}$ inch, while complete turns of the screw are obtained from a scale F on the left, which also appears in the field of view. In order to calibrate the horizontal scale the Nernst lamp and nitro-cellulose beam are removed, the Nicols rotated to parallelism, and a beam of solar light focussed on to the slit. The position of lines of known wave-length are noted with reference to the horizontal scale, and from these observations the constants in the equation

$$x_1 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

are found for calibrating the positions of the black bands.

In the observations it is found that the depth of the beam does not appear quite constant throughout the field, an error due to the combined imperfections of the lenses and prisms employed. The maximum change of depth is about 2 per cent., and a correction is therefore necessary to reduce all vertical distances to a constant depth of beam.

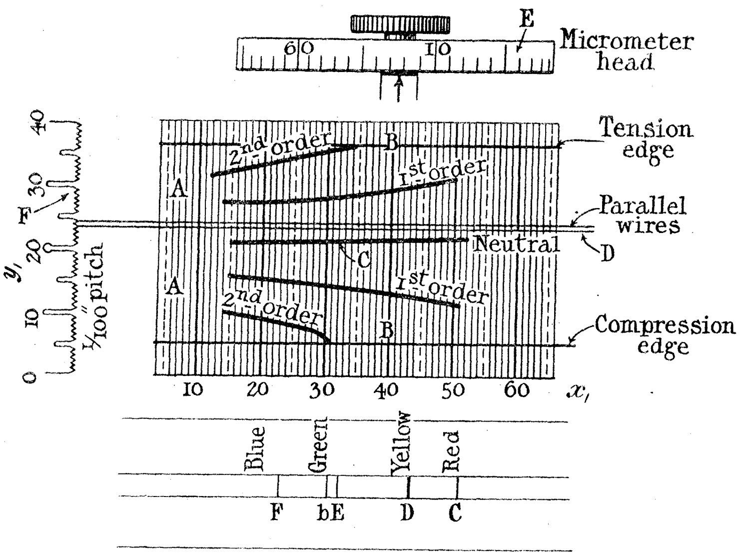


Fig. 12. View of spectrum.

The observations made are too numerous to give in detail, but typical examples of some measurements are shown in the accompanying fig. 13, in which the black bands, due to extinction of light, are drawn for a bending moment of 178·3 in pound and inch units. This, however, is not the exact appearance of the bands in the field of view owing to variation in the wave-length which alters the horizontal scale, but is here made uniform for plotting.

For some calculations, however, it is more convenient to show the form of the bands corresponding to a definite wave-length with a varying bending moment. Owing to the presence of a small amount of initial retardation in plates of nitro-cellulose, due to the method of manufacture, which leaves traces of initial stress, there is generally some slight difference between the bands on each side of the neutral axis, and a more accurate value is probable if the mean value for the two sides is taken.

If relative retardation is a linear function of the stress difference, these new abscissæ will represent the mean stress, but if the strain varies linearly they will also represent strains to another scale.

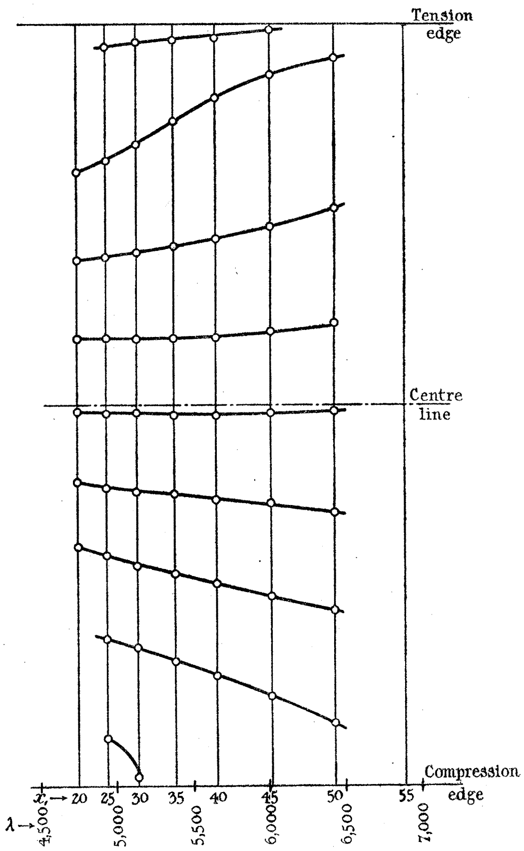


Fig. 13. Stress bands in spectrum load – 18 lb. B.mt. 178·3 lb.-inches.

If then the mean distances of the bands are plotted as ordinates against the order of the band as abscissæ, fig. 14, a convenient form of diagram OCF is obtained in

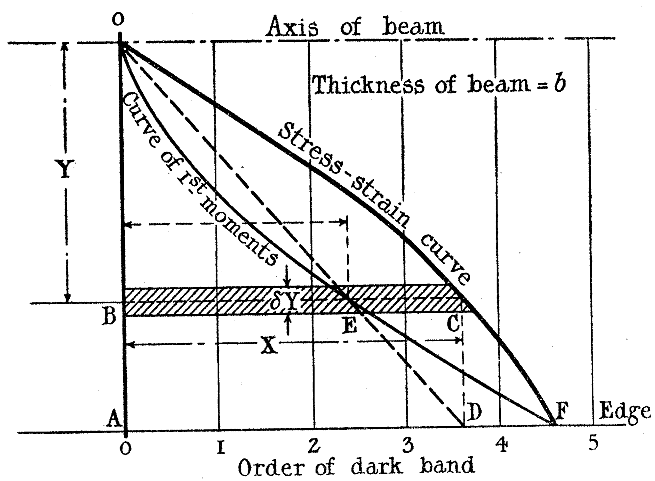


Fig. 14.

rectangular co-ordinates X, Y, in which Y is the distance from the neutral axis to a scale α and f is the stress to a scale β, or

$$y = \mathrm{Y} \,.\, \alpha \,; \qquad f = \mathrm{X} \,.\, \beta, \text{ say.}$$

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Now the bending moment

$$M = \int_{-\frac{1}{2}d}^{+\frac{1}{2}d} fb \, . \, y \, . \, dy$$

for a breadth $= b$ and a depth $= d$, or

$$M = b \, . \, \alpha^2 \, . \, \beta \int_{-\frac{1}{2}d}^{+\frac{1}{2}d} X \, . \, Y \, . \, dY$$

$= 2 \, . \, \alpha^2\beta \, . \, b \times$ first moment of the area of the diagram about the neutral axis.

If now any point C on the curve is projected on to the edge line at D and the line OD is drawn to the origin of co-ordinates intersecting the horizontal through C at E, then

$$BE : Y :: AD : AO = X : \frac{d}{2},$$

or

$$XY = BE \times \frac{d}{2}.$$

Hence $\int_{-\frac{1}{2}d}^{\frac{1}{2}d} X \, . \, Y \, . \, dY$ represents the area OEFAO $\times d$.

A typical example of one of these diagrams is shown in the accompanying fig. 15,

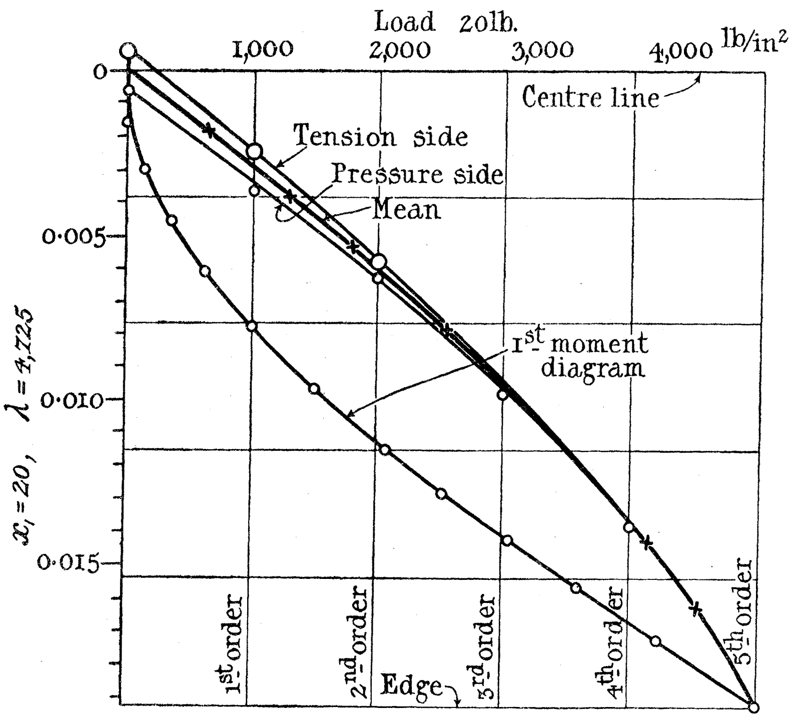

Fig. 15. Stress-strain curves.

of which about forty were actually prepared. The first moment areas M′, as determined by planimeter measurements, are shown in Table III., and when these are divided by the bending moment a value of $\alpha^2\beta b$ is obtained. If, however, the relative

TABLE III.—First Moments of Stress-strain Curves = M′.

x_1.	λ.	W = 20 lb.	18 lb.	16 lb.	14 lb.	12 lb.	10 lb.	9 lb.
		M = 198 lb./in.	178·3 lb./in.	158·7 lb./in.	139·2 lb./in.	119·6 lb./in.	100·0 lb./in.	90·2 lb./in.
20	4725	46·20	40·30	33·50	29·65	25·60	—	—
25	4925	43·15	38·40	32·15	28·65	24·35	—	—
30	5135	41·25	36·50	31·80	27·00	23·35	19·95	17·90
35	5380	39·85	36·40	31·10	25·75	21·20	18·25	—
40	5660	38·00	34·65	28·65	24·00	20·55	17·50	—
45	6015	36·65	31·90	27·85	23·00	19·25	17·00	—
50	6430	33·50	29·55	25·00	21·15	18·60	15·35	—

retardation is assumed to be independent of the wave-length, the mean value of $\frac{M'\lambda}{M}$ affords values of β corresponding to different wave-lengths. The values of β determined in this way are shown in the accompanying Table IV.

TABLE IV.

x_1.	λ.	β.
20	4725	920
25	4925	960
30	5135	1000
35	5380	1050
40	5660	1104
45	6015	1172
50	6430	1255

In order to determine the scale of strains the value of YOUNG's modulus E = 309,000, as taken from the measurements in tension within the elastic limit, is assumed to hold near the neutral axis of the beam for all loads, and since in this region we have the strain $e = \text{Y} \,.\, s$, where s is the scale for strains, then

$$\text{E} = \frac{df}{de} = \frac{\beta}{s} \cdot \frac{d\text{X}}{d\text{Y}},$$

or

$$s = \frac{\beta}{\text{E}} \cdot \frac{d\text{X}}{d\text{Y}}.$$

The slopes $\frac{d\text{X}}{d\text{Y}}$ near the neutral axis are measured from the diagrams similar to those of fig. 15, and their values are shown in the accompanying Table V.

multiplied by the values of β appropriate to the wave-length. Their mean values afford measures of the strains as the table shows.

TABLE V.—Values of $\beta \cdot \frac{\delta X}{\delta Y}$.

x_1 = 20	1165	1012	805	681	589	—	—
= 25	1167	1000	810	672	581	—	—
= 30	1180	1000	810	670	572	473	435
= 35	1180	998	840	657	559	466	—
= 40	1196	1032	848	657	563	468	—
= 45	1201	996	838	656	563	479	—
= 50	1205	997	831	677	564	467	—
Total . . .	8294	7035	5782	4670	3991	2353	435
Mean . . .	1185	1005	826	667	570	471	435 ?
Scale of strains $s = \frac{1}{E} \cdot \beta \cdot \frac{\delta X}{\delta Y}$.	$3 \cdot 84 \times 10^{-3}$	$3 \cdot 25 \times 10^{-3}$	$2 \cdot 67 \times 10^{-3}$	$2 \cdot 16 \times 10^{-3}$	$1 \cdot 85 \times 10^{-3}$	$1 \cdot 52 \times 10^{-3}$	$1 \cdot 39 \times 10^{-3}$
$\frac{1}{s} \times 10^{-3}$	0·260″	0·308″	0·375″	0·463″	0·541″	0·658″	0·720″

The data afforded by this method is therefore sufficient to construct a stress-strain curve entirely from these measurements of the bands due to retardation in the spectrum, and if the assumptions are correct, it ought to agree with a similar diagram constructed from data obtained independently. The stress-strain diagram

TABLE VI.—Stress-strain Values.

	x_1.	λ.	Strains.									
			0·002.	0·004.	0·006.	0·008.	0·010.	0·012.	0·014.	0·016.	0·018.	0·0192.
W = 20 lb.	20	4725	596	1210	1815	2350	2834	3290	3700	4090	4420	4570
	25	4925	612	1205	1827	2362	2830	3290	3672	4015	4340	4530
	30	5135	620	1230	1840	2360	2810	3250	3620	3950	4220	4340
	35	5380	629	1258	1855	2390	2850	3280	3644	3980	4270	4400
	40	5660	631	1262	1880	2424	2900	3330	3730	4020	4265	4360
	45	6015	647	1271	1860	2420	2910	3255	3765	4130	4450	4610
	50	6430	654	1295	1910	2440	2880	3295	3695	4000	4260	4370
Mean . . .			627	1247	1855	2394	2859	3284	3689	4026	4311	4454

obtained from spectrum observations gives the following values, of which Table VI. is a typical example, from which the mean values are obtained as follows :—

Strains	0·001	0·002	0·003	0·004	0·005	0·006	0·007	0·008
Stresses in lbs. per sq. inch	310	620	920	1240	1540	1845	2135	2400
Strains	0·009	0·010	0·011	0·012	0·013	0·014	0·018	0·0192
Stresses in lbs. per sq. inch	2665	2890	3055	3285	3460	3680	4310	4455

Table of mean values

Strain	0·001	0·002	0·003	0·004	0·005	0·006	0·007	0·008	0·009	0·010	0·011	0·012	0·013	0·014	0·016	0·018	0·0192
Stress	310	620	918	1,238	1,541	1,844	2,135	2,399	2,665	2,889	3,056	3,283	3,459	3,682	3,997	4,311	4,454
Corrected for thickness	310	621	919	1,240	1,544	1,849	2,141	2,407	2,675	2,901	3,070	3,300	3,478	3,703	4,023	4,343	4,490

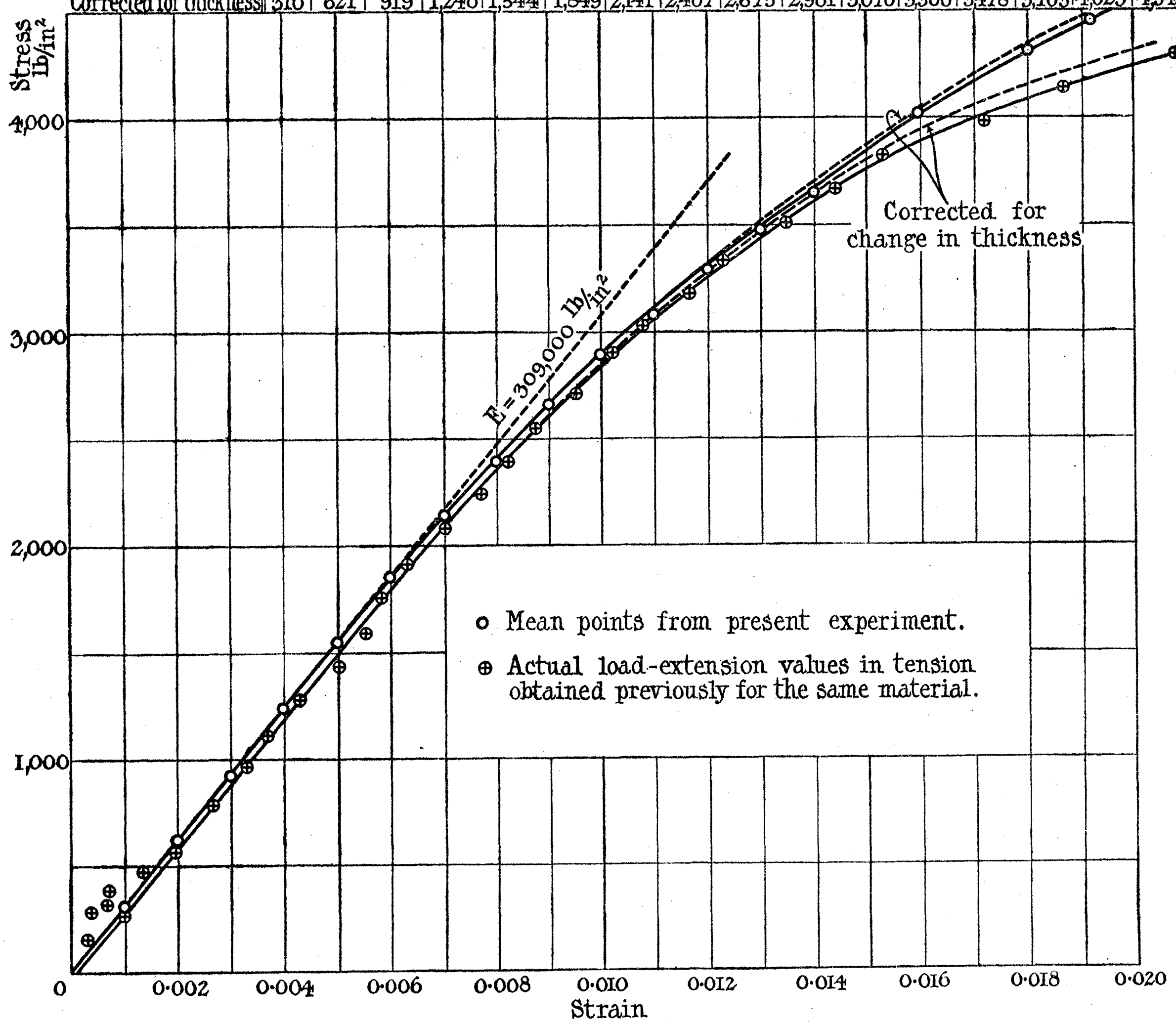

Fig. 16. Comparison of stress-strain curves.

This information has already been obtained however by observations on precisely similar material under direct tension stress, Table II. and fig. 13, and on comparing plots of the two sets of data obtained, fig. 16, the agreement is seen to be a remarkably close one up to about 3500 lbs. per sq. inch. This agreement is improved if the changes in thickness are allowed for, since the corrected curves then lie closer together, and strengthen the evidence in favour of the law of optical retardation being an effect of stress and not of strain, and also that it is still a linear function much beyond the elastic limit of the material.

The whole of the evidence, in fact, appears to show that the transparent nitrocellulose examined obeys a linear stress optical law which holds up to approximately twice the range of the elastic limit of stress; and that within this range optical determinations of stress distribution may be relied upon.

In conclusion we desire to express our grateful thanks for the help afforded in this work by the Department of Scientific and Industrial Research, also for valuable suggestions from Prof. Filon, F.R.S., and Prof. Porter, F.R.S., during its progress, and for the skilful assistance of Mr. F. H. Withycombe in preparing all the experimental apparatus required.

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